THE FLOW OF A VISCOUS, ELECTRICALLY CONDUCTING GAS IN A TRANSVERSE MAGNETIC FIELD IN THE PRESENCE OF HEAT TRANSFER

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This paper contains a generalization of the solution for plane flow presented in [1], in that it includes the effect of viscous dissipation.

The problem of the two-dimensional flow of an incompressible, viscous, electrically conducting gas through a channel formed by two parallel planes subjected to the action of a uniform magnetic field of intensity H_0, and in the presence of heat transfer from the walls, was considered in [1,2]; the first reference contained a solution for a uniform heat flux, but excluded viscous dissipation ; the second reference contained a solution which included the effect of viscous dissipation, but was restricted to the case of a wall of constant temperature.

We shall consider the flow of a fluid through a channel formed by two infinite, electrically non-conducting planes at $z = \pm b$, exposed to a normal uniform magnetic field H_0. The properties of the liquid are described by its electric conductivity σ, its density ρ, specific heat c_p, viscosity μ and thermal conductivity λ. The effect of temperature on these properties will be ignored.

If the walls are impermeable to the fluid, it is possible to consider that the streamlines are confined in planes $x - y$. We choose the axis y to make the velocity component $W_y = 0$. Then the solution can be written

$$W_x = W(z), \qquad W_y = 0, \qquad W_z = 0,$$
$$H_x = H_x(z), \qquad H_y = 0, \qquad H_z = H_0, \qquad p = p(x, z), \qquad T = T(x, z) \tag{1}$$

the remaining parameters being constant.

The system of equations satisfied by solution (1) has the form

$$\frac{\partial}{\partial x}\left(p+\mu_e\frac{H^2}{2}\right)=\mu\frac{d^2W}{dz^2}+\mu_e H_0\frac{dH_x}{dz}\qquad\left(\alpha=\frac{\lambda}{\rho c_p}\right)$$

$$\frac{\partial}{\partial z}\left(p+\mu_e\frac{H^2}{2}\right)=0,\qquad H_0\frac{dW}{dz}+\frac{1}{\mu_e\sigma}\frac{d^2H_x}{dz^2}=0 \tag{2}$$

$$W\frac{\partial T}{\partial x}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{1}{\rho c_p\sigma}\left(\frac{dH_x}{dz}\right)^2+\frac{\mu}{\rho c_p}\left(\frac{dW}{dz}\right)^2$$

Here α denotes the coefficient of heat transfer. The first three equations yield the following solution for W and H_x [3]:

$$W=\frac{PN}{\mu_e{}^2\sigma H_0{}^2}\frac{\cosh N-\cosh(Nz/b)}{\sinh N},\qquad H_x=\frac{Pb}{\mu_e H_0}\left[\frac{\sinh(Nz/b)}{\sinh N}-\frac{z}{b}\right]$$

$$P=-\partial p/\partial x=\text{const},\qquad N=\mu_e H_0 b\sqrt{\sigma/\mu}$$

Here N denotes the Hartman number and b is the channel half-width.

In the present problem the distribution of heat sources is independent of x. It follows that the temperature profiles are similar in all cross-sections of the channel, and the temperature $T=\tau x+\theta(z)$. Thus, the fourth equation (2) can be reduced to the form

$$\frac{PN\tau}{\mu_e{}^2\sigma H_0{}^2}\left[\frac{\cosh N-\cosh(Nz/b)}{\sinh N}\right]= \tag{3}$$

$$=\alpha\frac{d^2\theta}{dz^2}+\frac{P^2}{\mu_e{}^2\sigma H_0{}^2\rho c_p}\left[\frac{N\cosh(Nz/b)}{\sinh N}-1\right]^2+\frac{\mu}{\rho c_p}\left[\frac{PN^2}{\mu_e{}^2\sigma H_0{}^2 b}\frac{\sinh(Nz/b)}{\sinh N}\right]^2$$

The value of τ can be found from the heat balance

$$\rho W^0 c_p b\tau=q+\int_0^b\left[\frac{1}{\sigma}\left(\frac{dH_x}{dz}\right)^2+\mu\left(\frac{dW}{dz}\right)^2\right]dz\quad\left(W^0=\frac{P}{\mu_e{}^2\sigma H_0{}^2}(N\coth N-1)\right) \tag{4}$$

where W^0 denotes the average velocity.

Solving (3) and (4) subject to $d\theta/dz=0$ for $z=0$ and $\theta=0$ for $z=\pm b$, we obtain

$$T=\tau x+\tau\frac{Pb^4}{2\alpha\mu N\sinh N}\left[\left(\frac{z}{b}\right)^2\cosh N-\frac{2}{N^2}\cosh\left(N\frac{z}{b}\right)-\frac{N^2-2}{N^2}\cosh N\right]-\frac{P^2b^4}{\alpha\rho c_p\mu N^2}$$

$$\left\{\frac{1}{4\,\mathrm{sh}^2N}\left[\frac{1}{2}\cosh\left(2N\frac{z}{b}\right)-\frac{1}{2}\cosh 2N+N^2\left(\frac{z^2}{b^2}-1\right)\right]-\frac{2\cosh(Nz/b)}{N\sinh N}+\right.$$

$$\left.+\frac{2\cosh N}{N\sinh N}+\frac{1}{2}\left(\frac{z^2}{b^2}-1\right)\right\}-\frac{P^2b^4N^2}{4\alpha\rho c_p\mu\sinh^2N}\left[\frac{1}{2}\cosh\left(2N\frac{z}{b}\right)-\frac{1}{2}\cosh 2N+\left(\frac{z^2}{b^2}-1\right)(N\cosh N-\sinh N)\right]$$

$$\theta=\frac{1}{\rho c_p bW^0}\left[q+\frac{P^2b^3}{\mu}\left(\frac{2N}{2N\sinh^2N}-\frac{1}{N^2}\right)\right]$$

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